Chapters 2/3: 1D/2D Kinematics
Thursday January 15th

- Review: Motion in a straight line (1D Kinematics)
- Review: Constant acceleration – a special case
- Chapter 3: Vectors
    - Properties of vectors
    - Unit vectors
    - Position and displacement
    - Velocity and acceleration vectors
- Constant acceleration in 2D and 3D
    - Projectile motion (next week)

Reading: up to page 36 in the text book (Ch. 3)

Summarizing

Displacement: $\Delta x = x_2 - x_1$

Average velocity: $v_{avg} = \bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{x_2 - x_1}{t_2 - t_1}$

Average speed: $s_{avg} = \bar{s} = \dfrac{\text{total distance}}{\Delta t}$

Instantaneous velocity:

$$v = \frac{dx}{dt} = \text{local slope of } x \text{ versus } t \text{ graph}$$

Instantaneous speed: magnitude of v

Summarizing

Average acceleration: $$a_{avg} = \bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

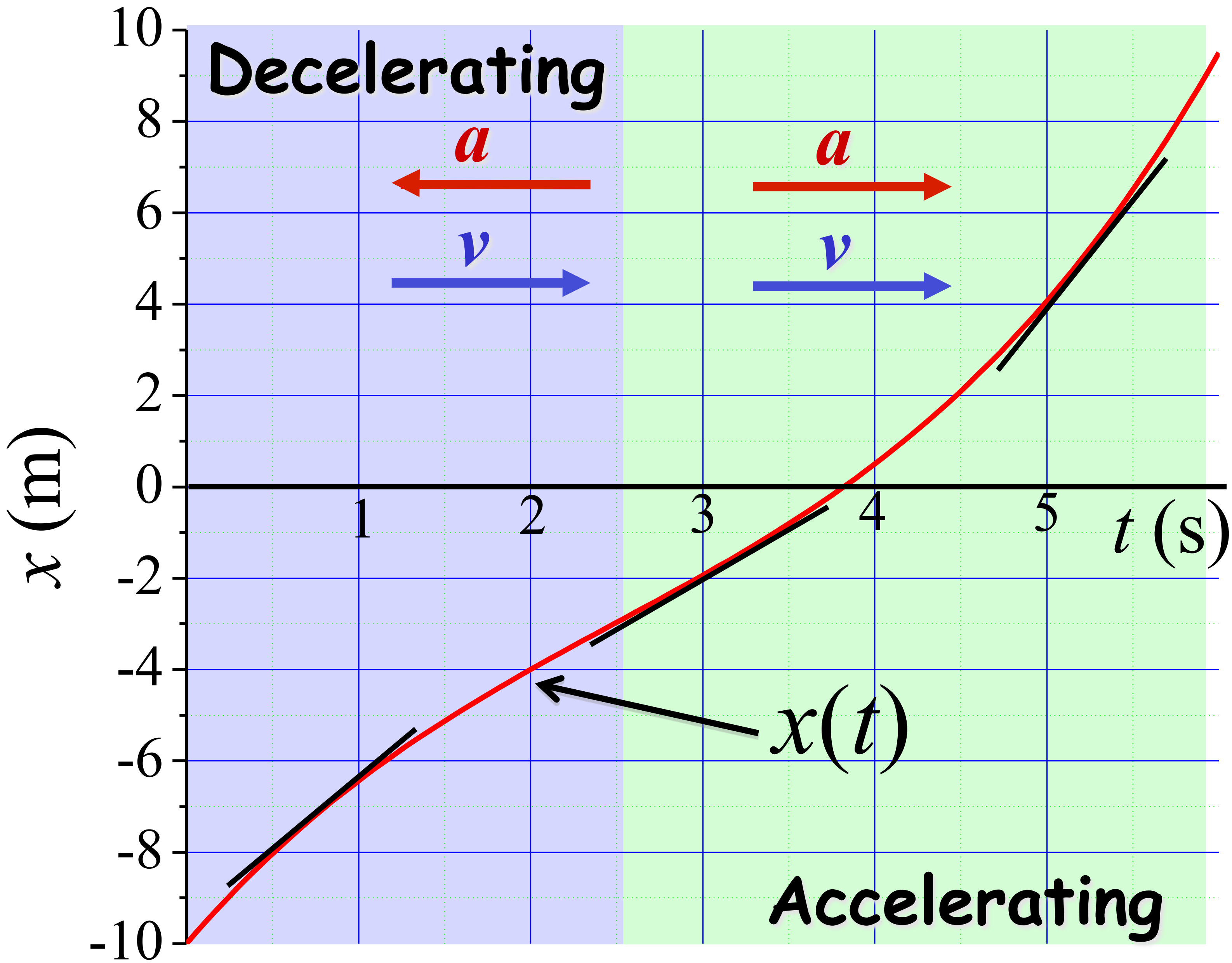
Instantaneous acceleration:

$$a = \frac{dv}{dt} = \text{local slope of } v \text{ versus } t \text{ graph}$$

In addition:

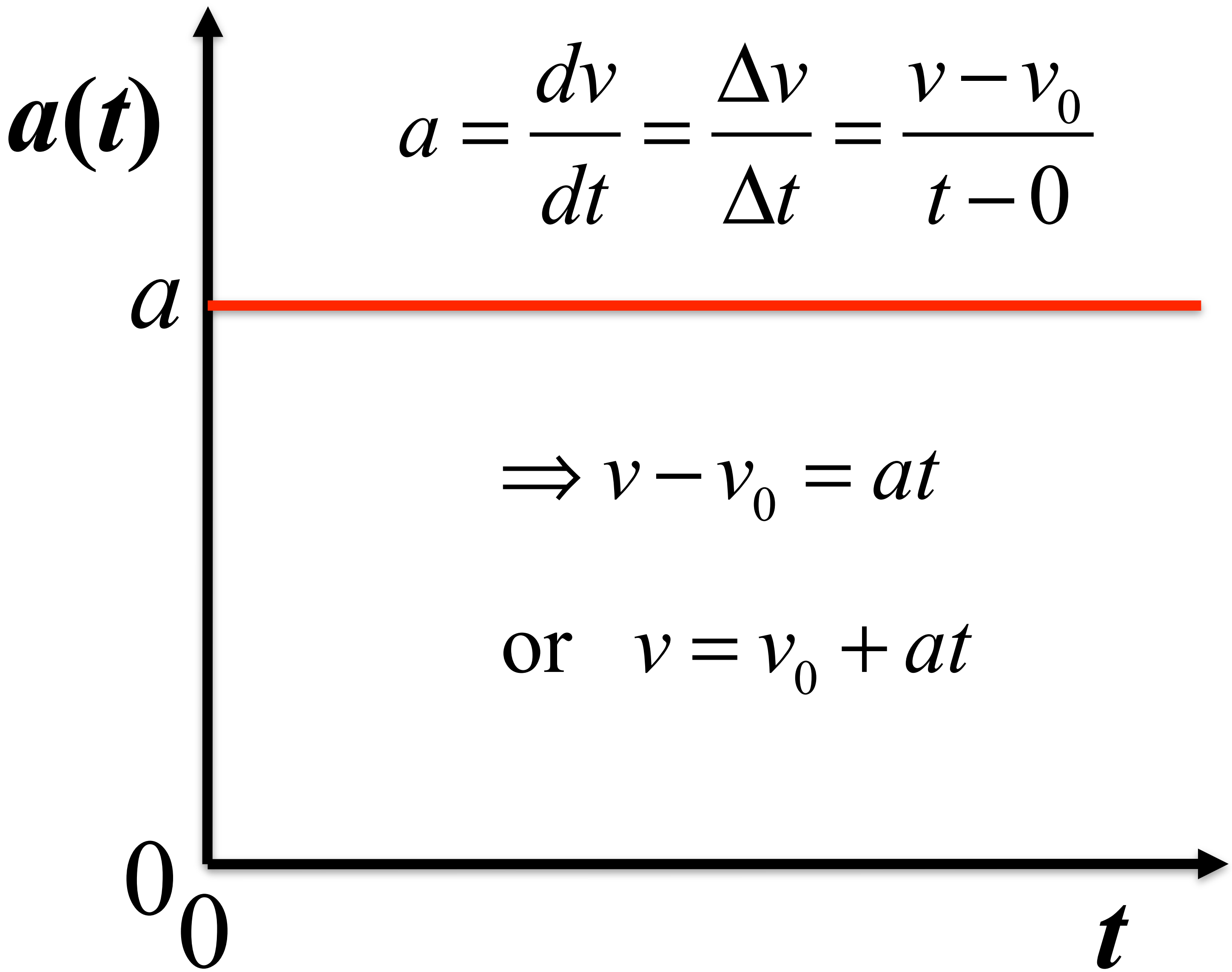
$$a = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} = \text{curvature of } x \text{ versus } t \text{ graph}$$

SI units for a are m/s^2 or $m.s^{-2}$



Decelerating
a
v
a
v
Accelerating
x(t)
x (m)
t (s)
10
8
6
4
2
0
-2
-4
-6
-8
-10
1
2
3
4
5

Constant acceleration: a special case

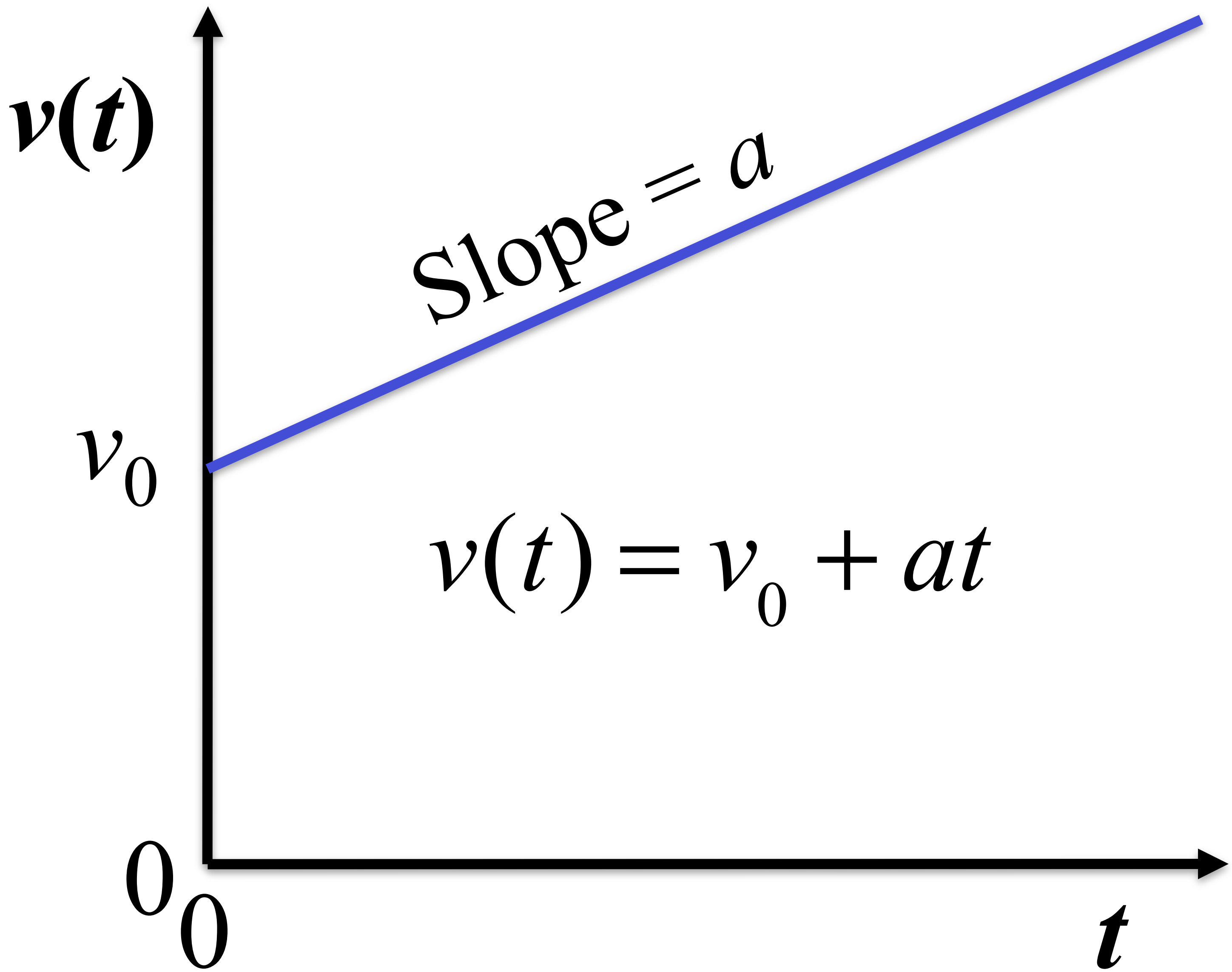


$$a = \frac{dv}{dt} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - 0}$$

$$\Rightarrow v - v_0 = at$$

$$\text{or} \quad v = v_0 + at$$

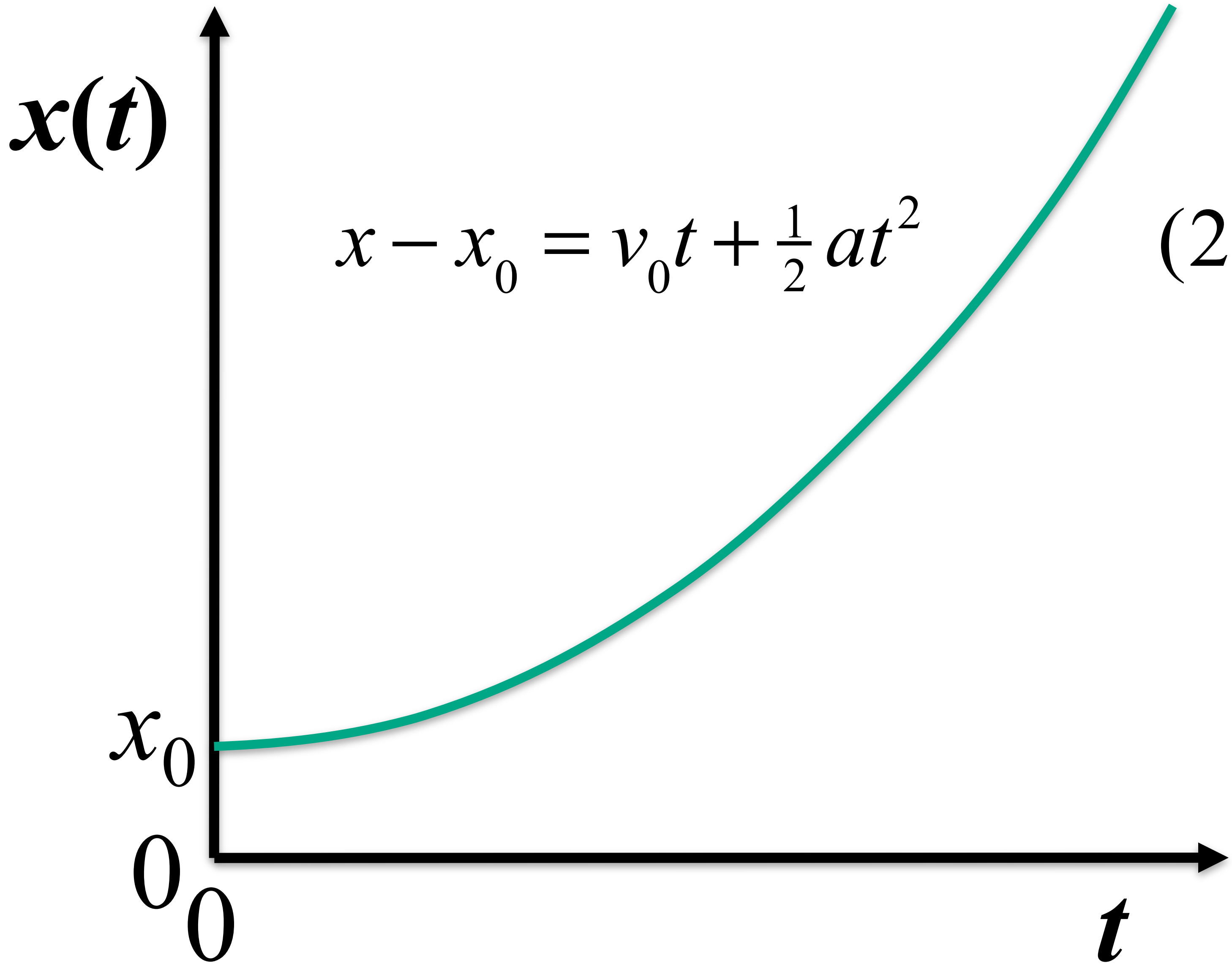
Constant acceleration: a special case



$$v(t) = v_0 + at \qquad (2.7)$$

Constant acceleration: a special case

$$x - x_0 = v_0 t + \tfrac{1}{2} a t^2 \qquad (2.10)$$



Equations of motion for constant acceleration

One can easily eliminate either a, t or v_0 by solving Eqs. 2-7 and 2-10 simultaneously.

Equation number	Equation	Missing quantity
2.7	$v = v_0 + at$	$x - x_0$
2.10	$x - x_0 = v_0 t + \frac{1}{2} a t^2$	v
2.11	$v^2 = v_0^2 + 2a(x - x_0)$	t
2.9	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
	$x - x_0 = vt - \frac{1}{2} a t^2$	v_0

Important: equations apply ONLY if acceleration is constant.

Equations of motion for constant acceleration

These equations work the same in any direction, e.g., along x, y or z.

Equation number	Equation	Missing quantity
2.7	$v_y = v_{0y} + a_y t$	$y - y_0$
2.10	$y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2$	v_y
2.11	$v_y^2 = v_{0y}^2 + 2a_y (y - y_0)$	t
2.9	$y - y_0 = \frac{1}{2}(v_{0y} + v_y)t$	a_y
	$y - y_0 = v_y t - \frac{1}{2} a_y t^2$	v_{0y}

Important: equations apply ONLY if acceleration is constant.

Equations of motion for constant acceleration

Special case of free-fall under gravity, $a_y = -g$.

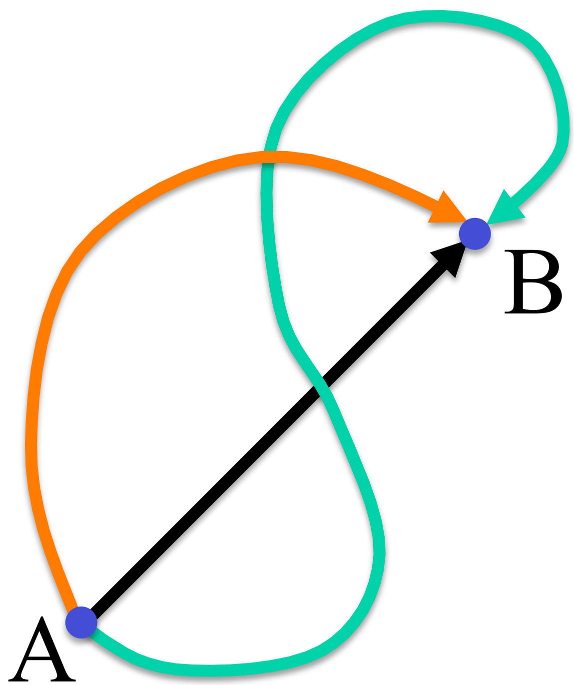
$g = 9.81\ \mathrm{m/s^2}$ here at the surface of the earth.

Equation number	Equation	Missing quantity
	$v_y = v_{0y} - gt$	$y - y_0$
	$y - y_0 = v_{0y}t - \frac{1}{2}gt^2$	v_y
	$v_y^2 = v_{0y}^2 - 2g(y - y_0)$	t
	$y - y_0 = \frac{1}{2}(v_{0y} + v_y)t$	a_y
	$y - y_0 = v_y t + \frac{1}{2}gt^2$	v_{0y}

Chapter 3: Introduction to Vectors

•A vector is a quantity that has both a magnitude and a direction, e.g., displacement, velocity, acceleration...

•Consider displacement as an example: if you travel from point A to B:



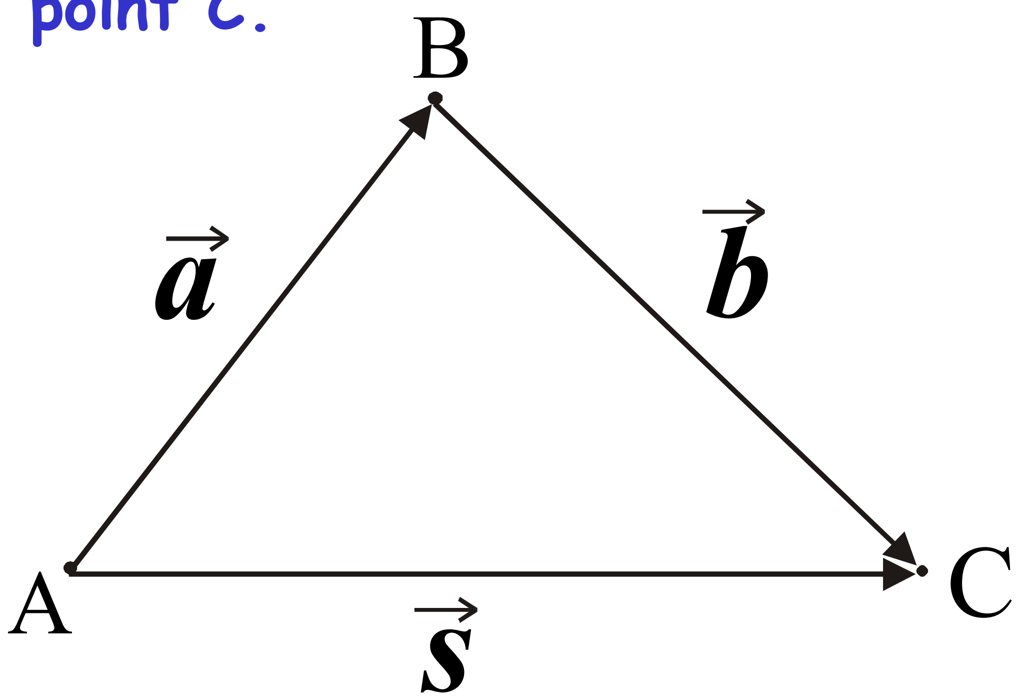
•It doesn't matter how you get from A to B, the displacement is simply the straight arrow from A to B.

•All arrows that have the same length and direction represent the same vectors, i.e. a vector is invariant under translation.

Adding vectors geometrically

- Note: overhead arrow is used to denote a vector quantity.
- If you travel from point A to point B, and then from point B to point C, your resultant displacement is the vector from point A to point C.

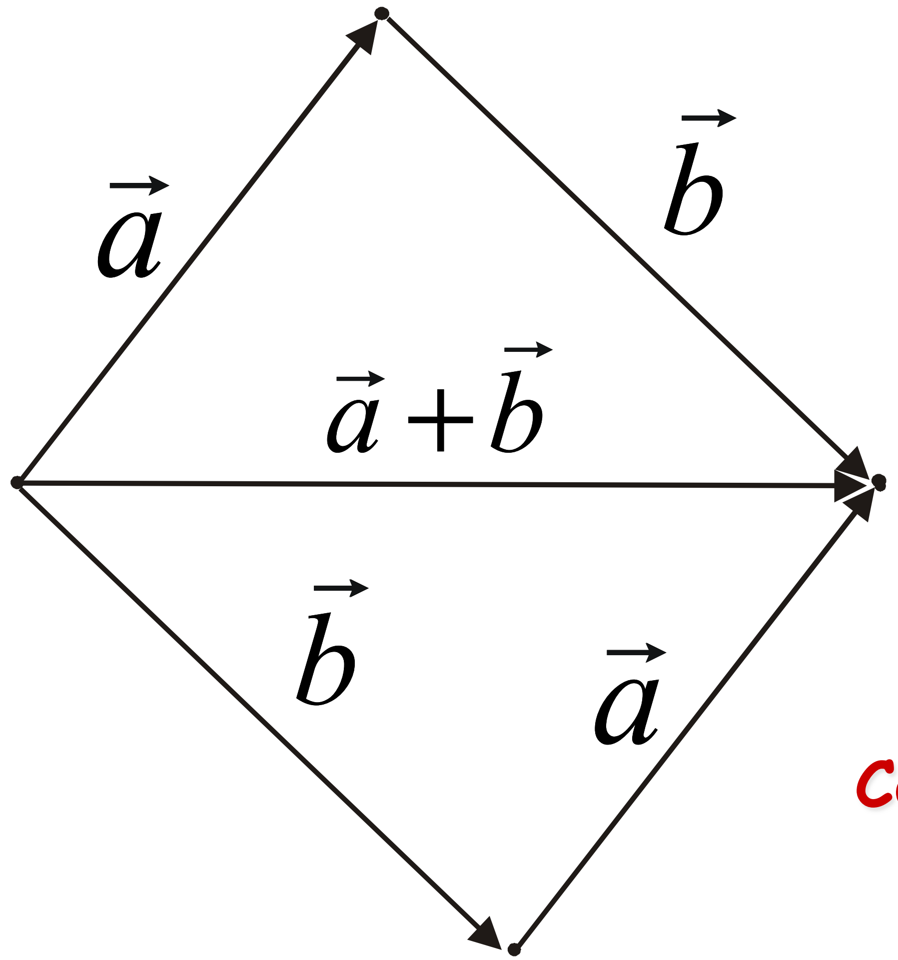
$$\vec{s} = \vec{a} + \vec{b}$$



- Vectors are added graphically by placing the tail of one vector at the head of the other.

Rules for vector addition

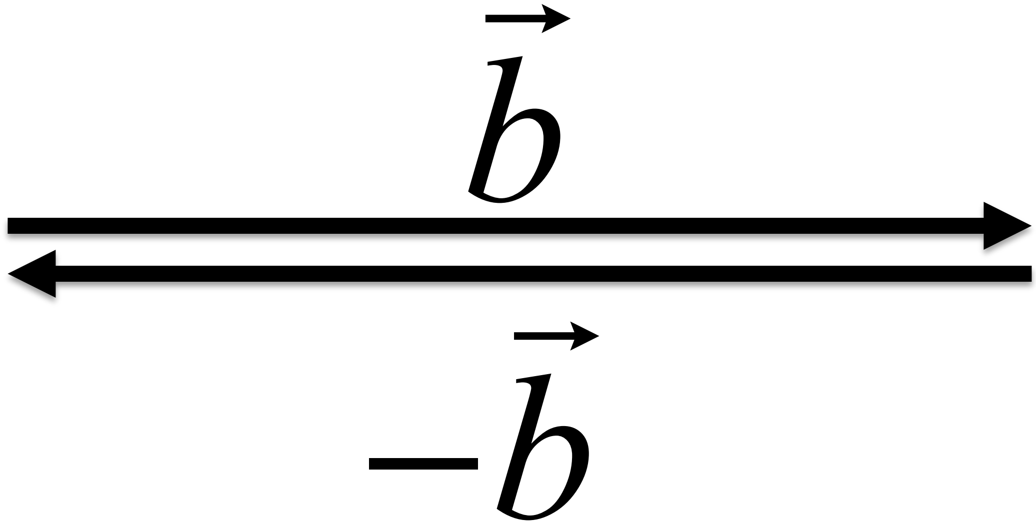
- In spite of the fact that vectors must be handled mathematically quite differently from scalars, the rules for addition are quite similar.



$$\vec{a}+\vec{b}=\vec{b}+\vec{a}$$

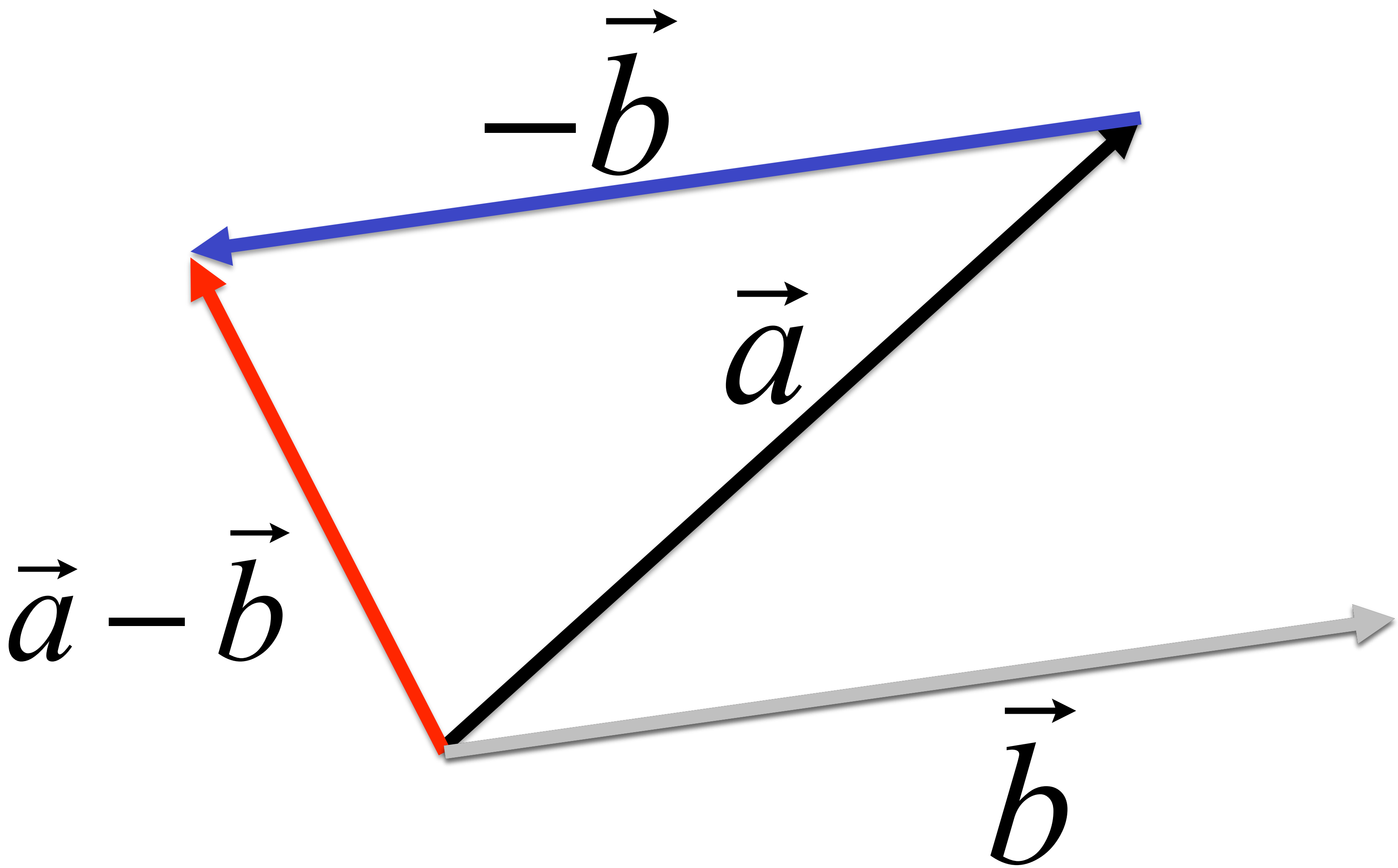
Commutative law

Vector subtraction



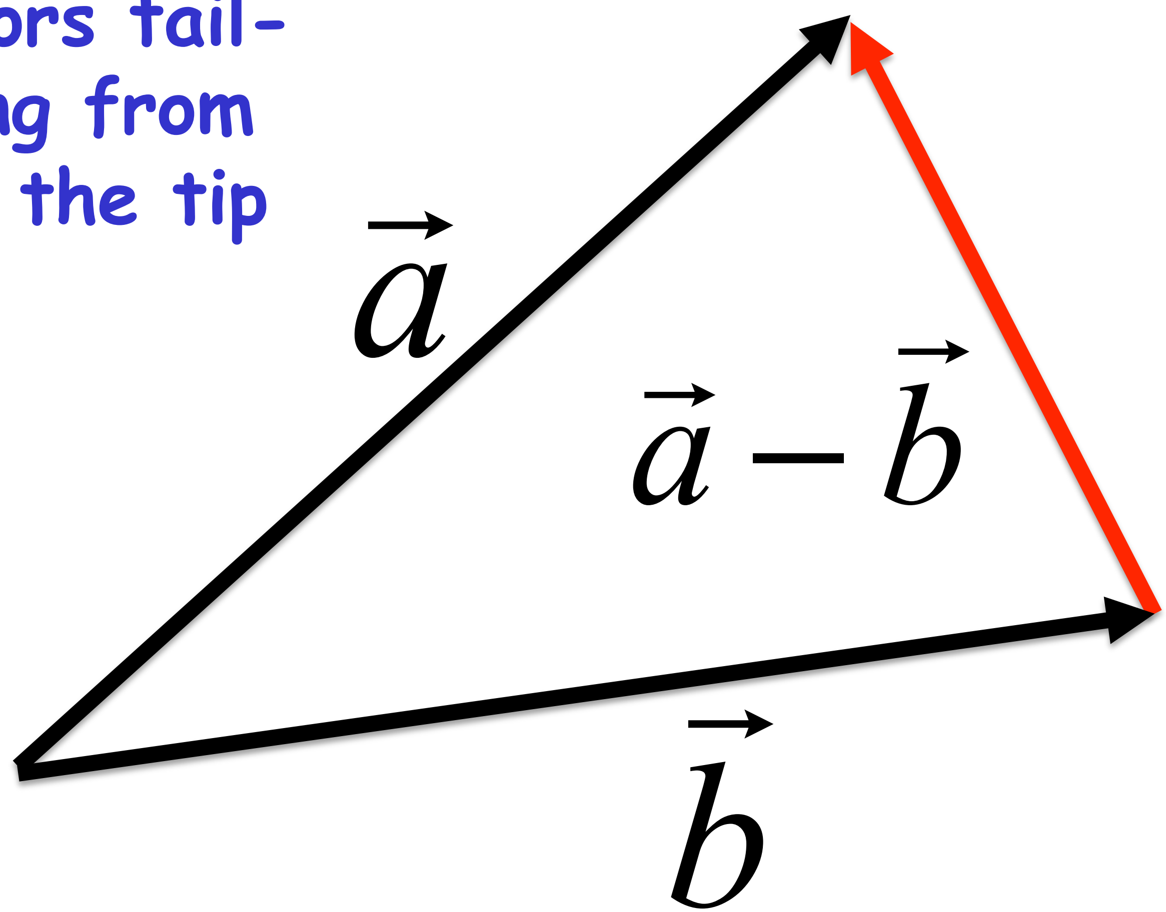
$$\vec{b}+\left(-\vec{b}\right)=\vec{b}-\vec{b}=0$$

Vector subtraction

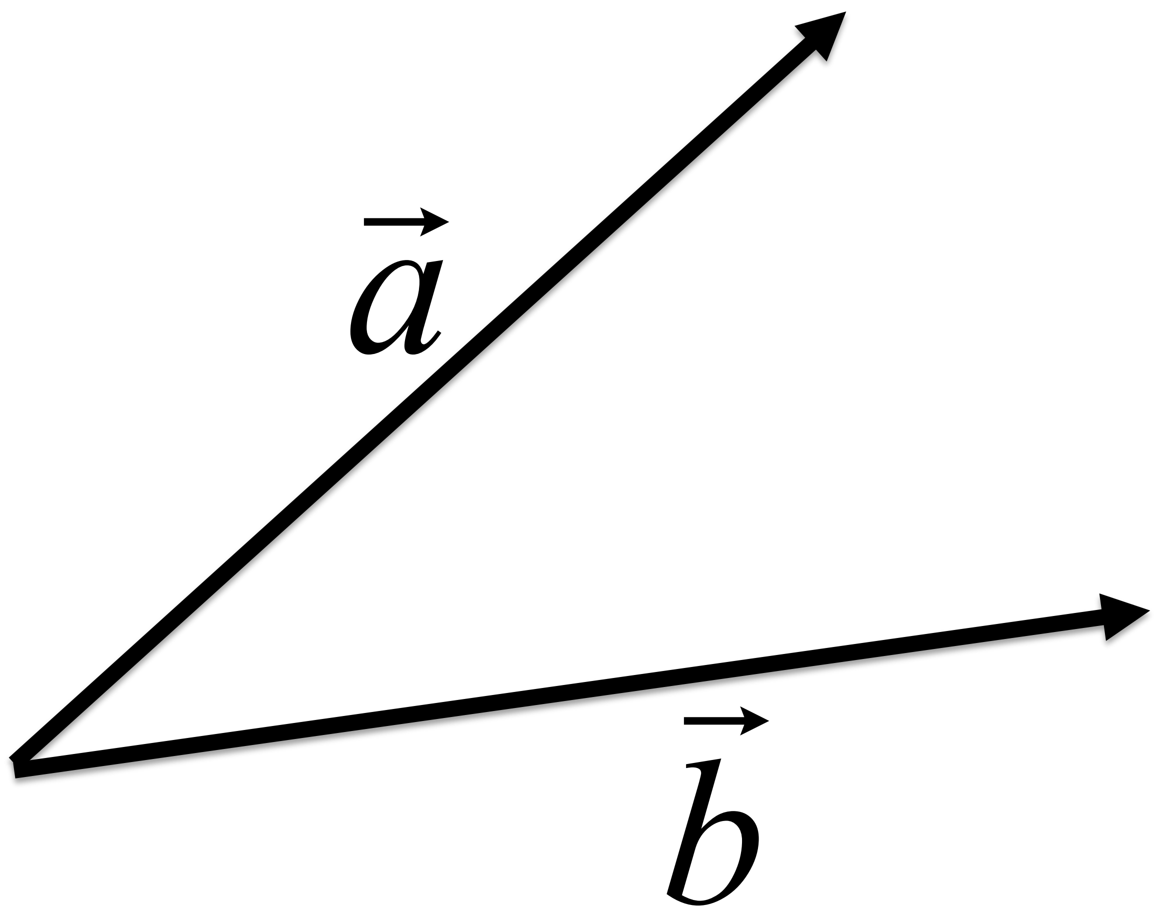


Vector subtraction

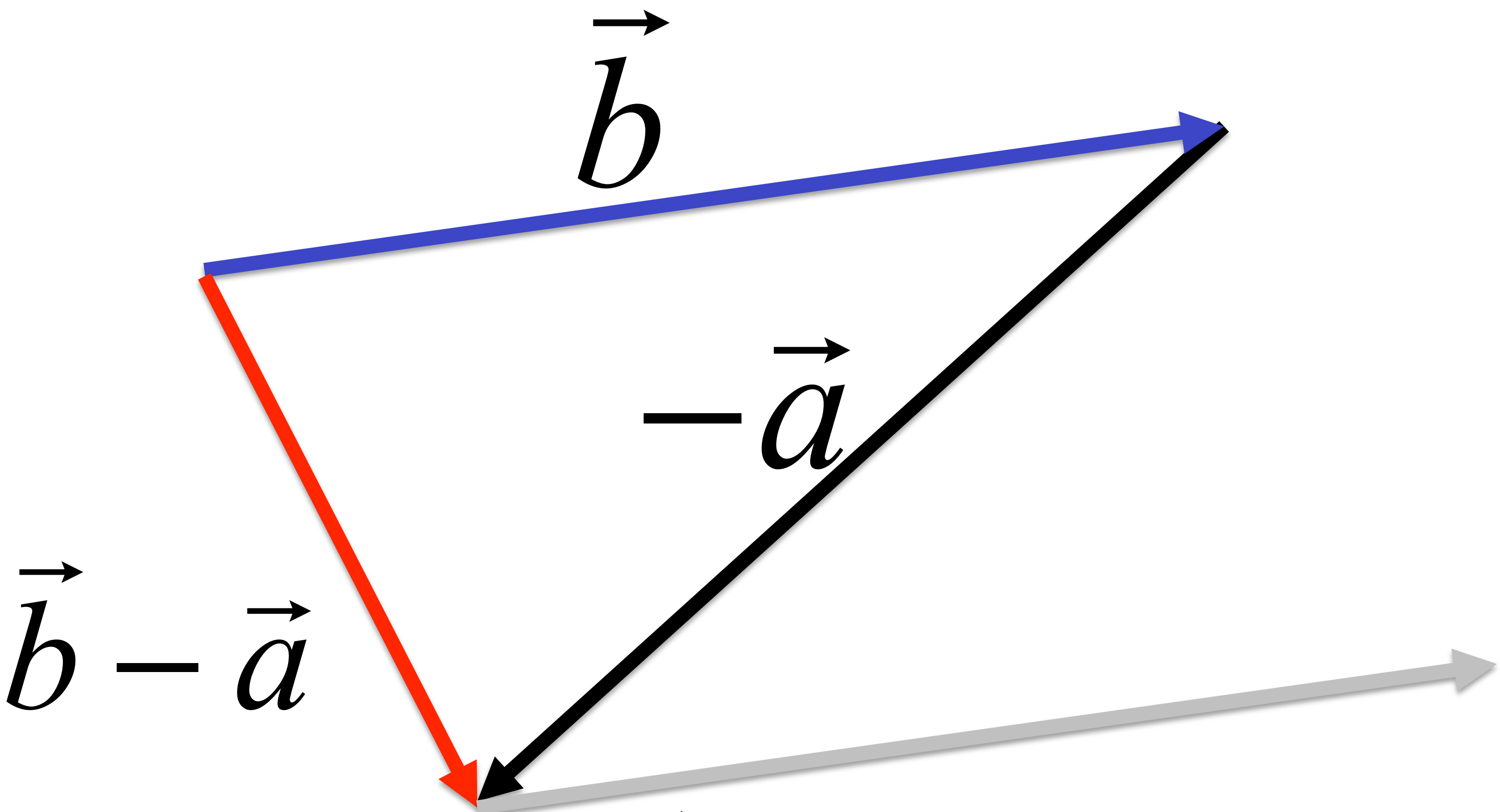
This will be important later: this is equivalent to putting vectors tail-to-tail and going from the tip of $\vec{b}$ to the tip of $\vec{a}$.



Vector subtraction



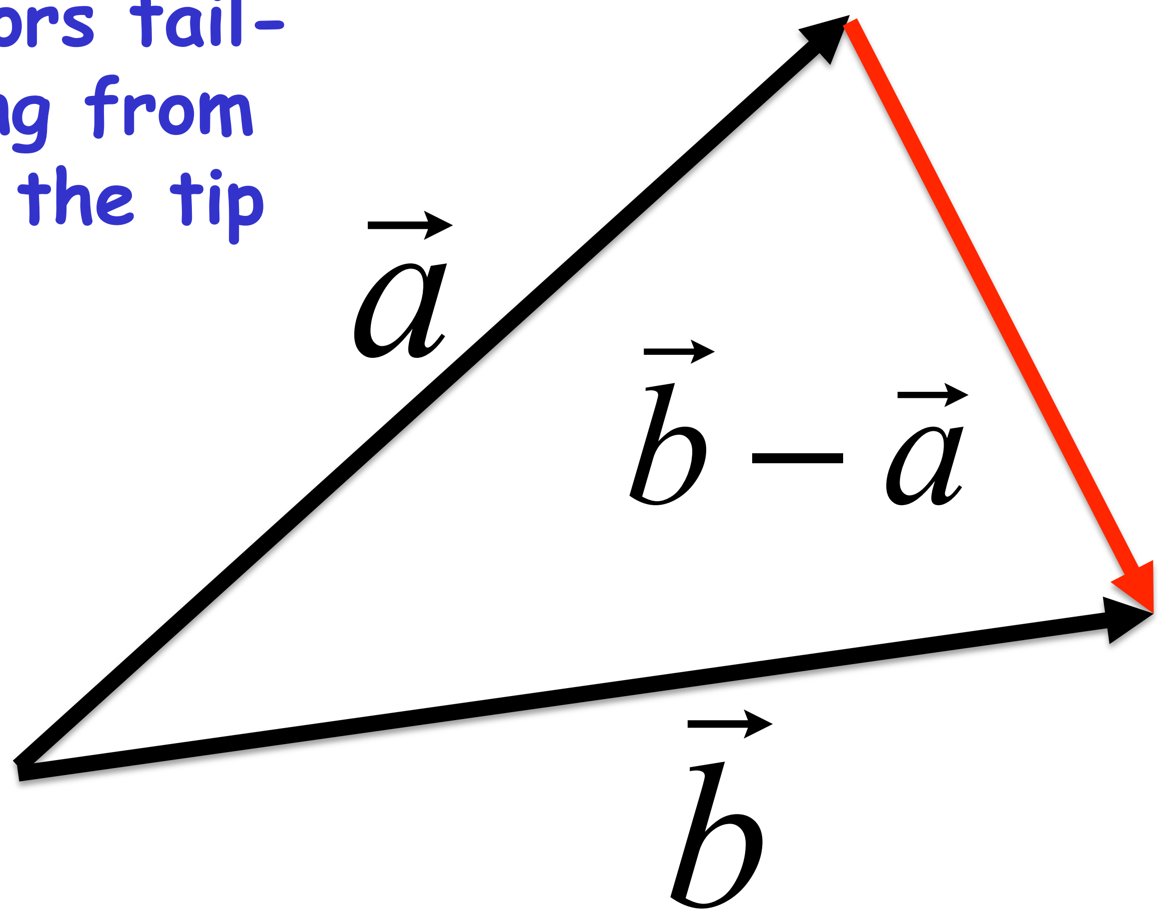
Vector subtraction



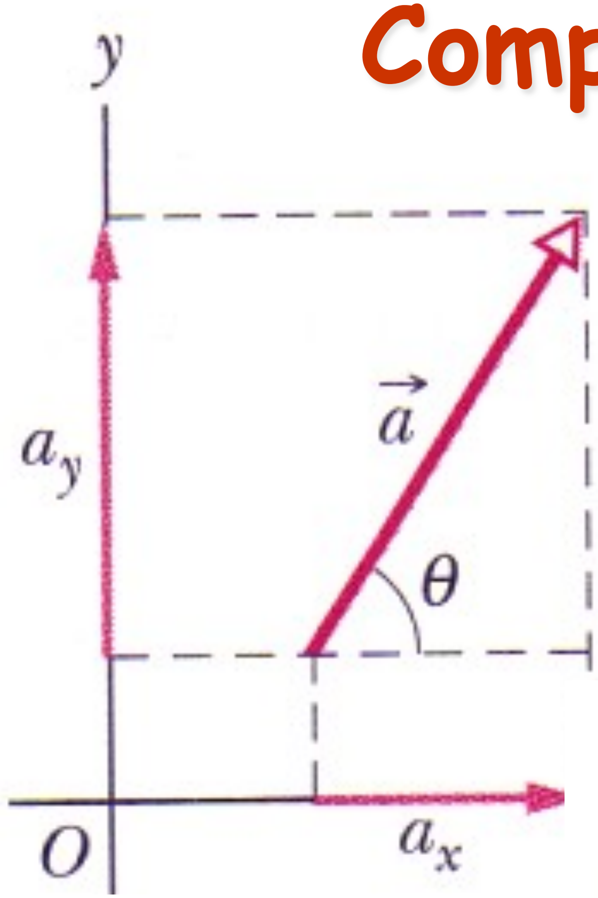
$$\vec{b} - \vec{a} = -(\vec{a} - \vec{b})$$

Vector subtraction

This will be important later: this is equivalent to putting vectors tail-to-tail and going from the tip of $\vec{a}$ to the tip of $\vec{b}$.



Components of vectors

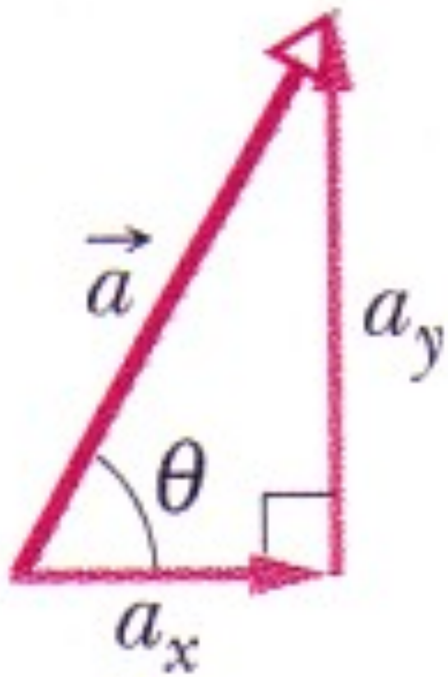


Resolving vector components

$$a_x = a\cos\theta$$

$$a_y = a\sin\theta$$

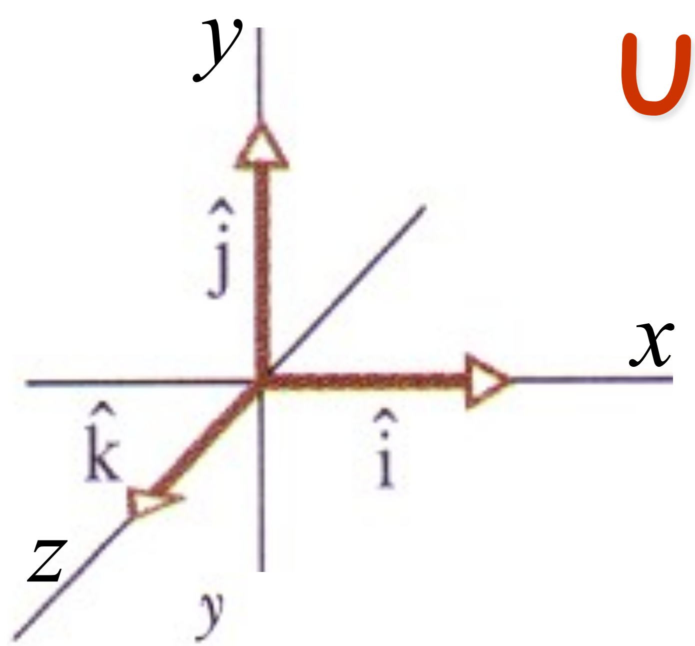
The inverse process



$$a = \sqrt{a_x^2 + a_y^2}$$

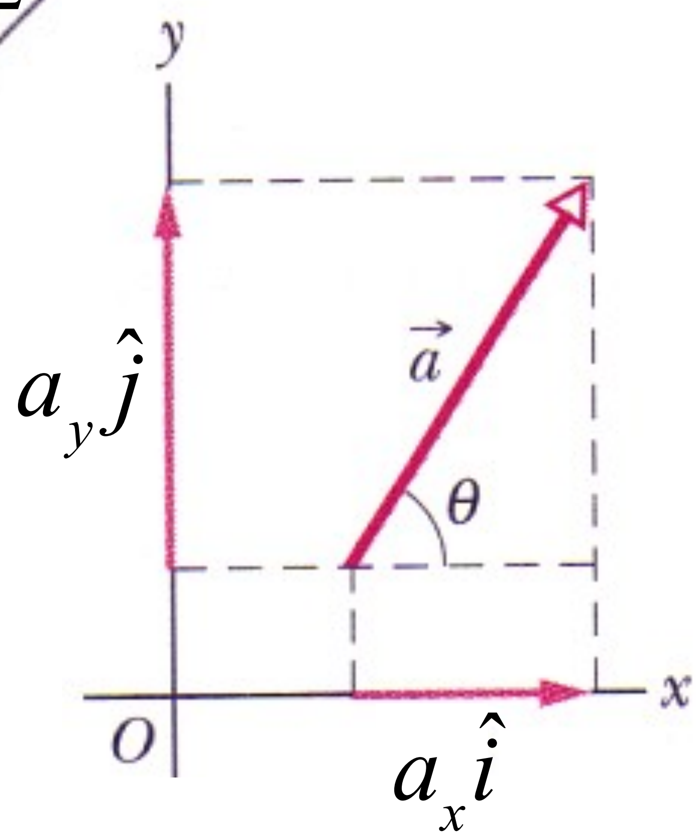
$$\tan\theta = \frac{a_y}{a_x}$$

Unit vectors



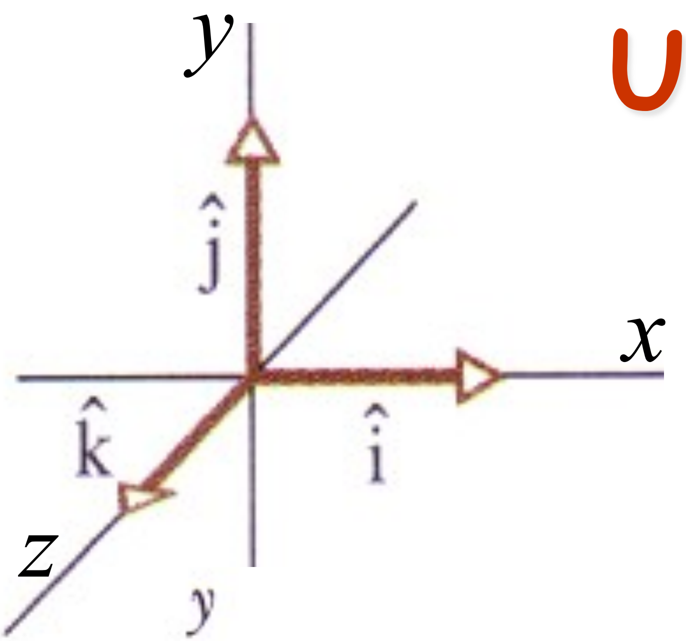
$\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors

They have length equal to unity (1), and point respectively along the x, y and z axes of a right handed Cartesian coordinate system.



$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

Unit vectors

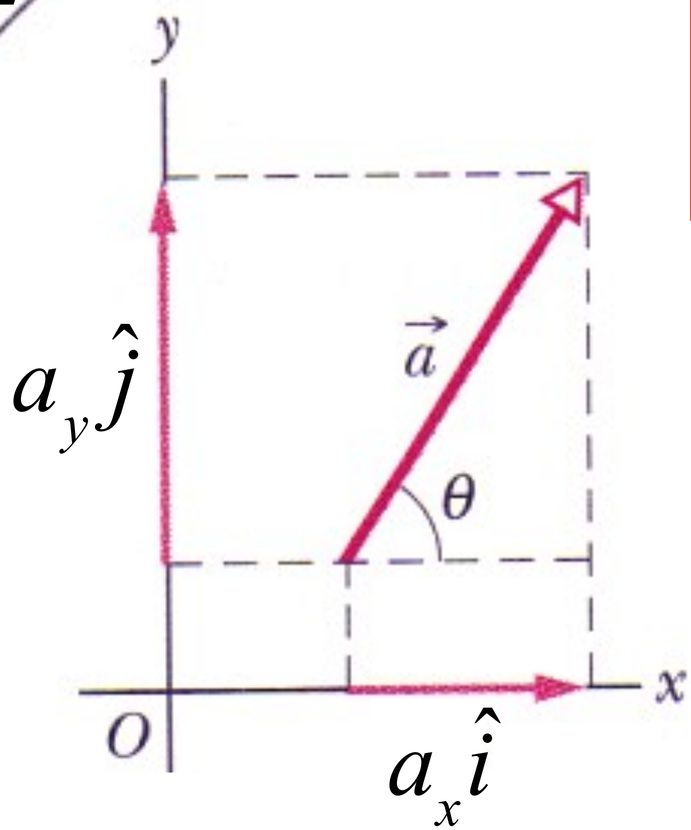


$\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors

Important Note:

Book uses: $\hat{i}, \hat{j}, \hat{k}$

LONCAPA uses: $\hat{x}, \hat{y}, \hat{z}$



$$\vec{a} = a\cos\theta\hat{i} + a\sin\theta\hat{j}$$

Note: θ is **usually** measured from x to y (in a right-handed sense around the z-axis)

Unit vectors

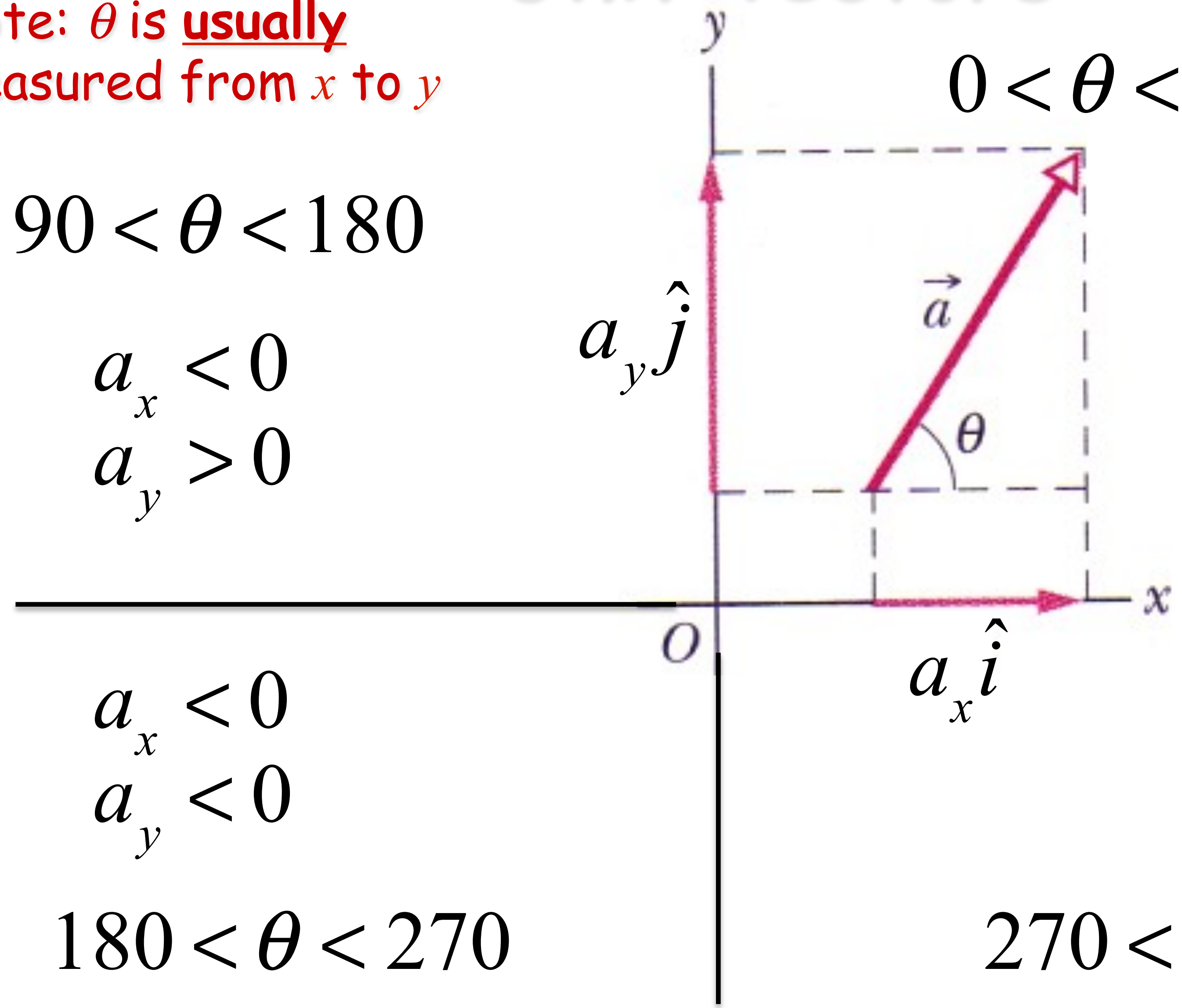
Note: θ is **usually** measured from x to y

$0 < \theta < 90$

$a_x > 0$
$a_y > 0$

$90 < \theta < 180$

$a_x < 0$
$a_y > 0$



$a_x < 0$
$a_y < 0$

$180 < \theta < 270$

$a_x > 0$
$a_y < 0$

$270 < \theta < 360$

$$\vec{a} = a\cos\theta\,\hat{i} + a\sin\theta\,\hat{j}$$

Adding vectors by components

Consider two vectors:

$$\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

&

$$\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$$

Then…

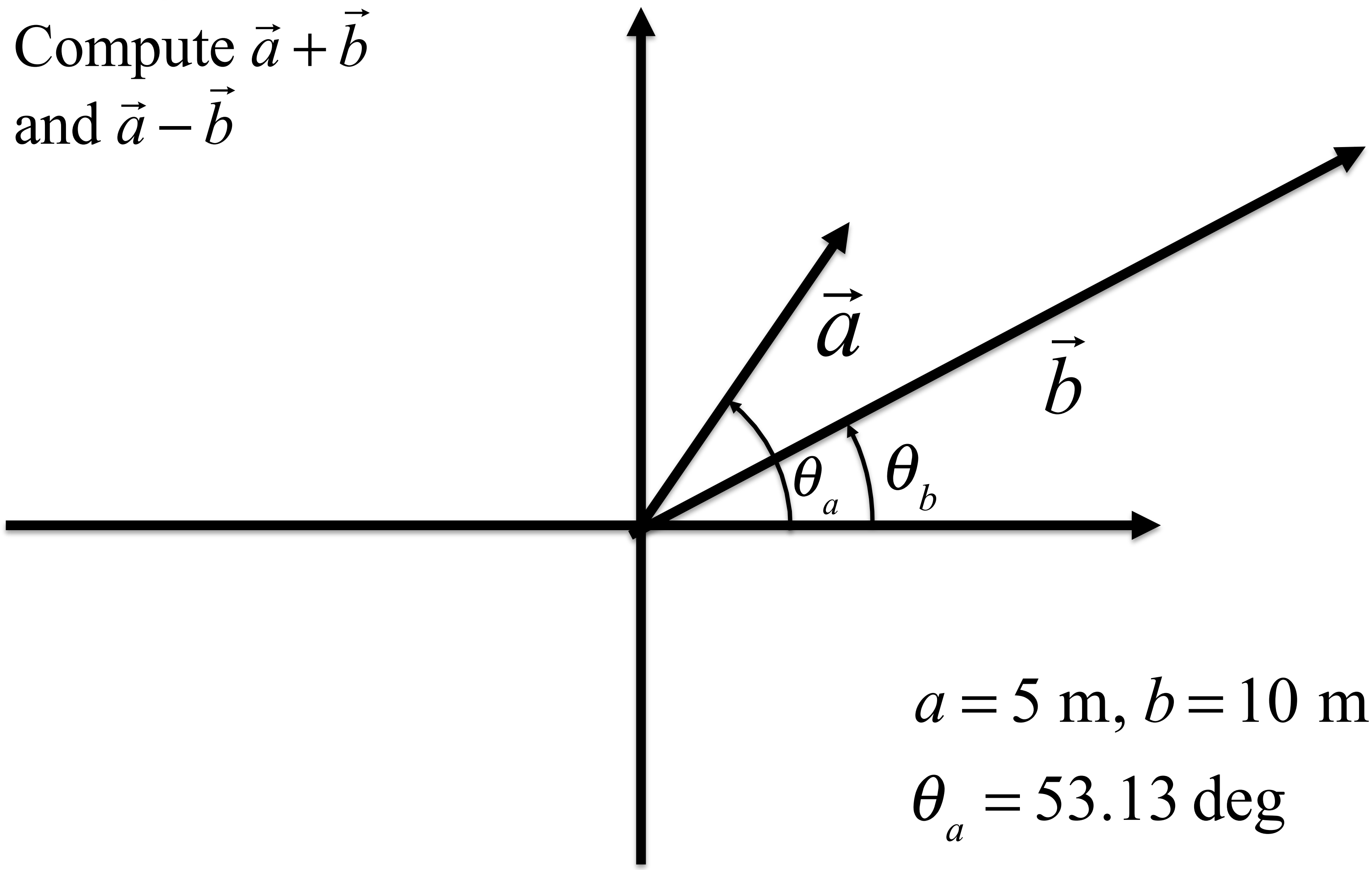
$$\Delta\vec{r}_{1\to 2} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

&

$$\vec{r}_1 + \vec{r}_2 = (x_2 + x_1)\hat{i} + (y_2 + y_1)\hat{j} + (z_2 + z_1)\hat{k}$$

Example:

Compute $\vec{a}+\vec{b}$
and $\vec{a}-\vec{b}$



$a = 5 \text{ m}, \ b = 10 \text{ m}$

$\theta_a = 53.13 \text{ deg}$

$\theta_b = 36.87 \text{ deg}$

Appendices

The scalar product in component form

$$\vec{a}\cdot\vec{b}=\left(a_x\hat{\mathrm{i}}+a_y\hat{\mathrm{j}}+a_z\hat{\mathrm{k}}\right)\cdot\left(b_x\hat{\mathrm{i}}+b_y\hat{\mathrm{j}}+b_z\hat{\mathrm{k}}\right)$$

$$\vec{a}\cdot\vec{b}=a_xb_x+a_yb_y+a_zb_z$$

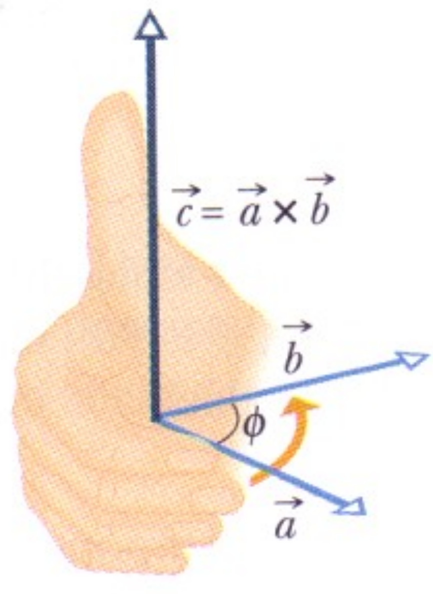
Because:

$$\hat{i}\cdot\hat{i}=\hat{j}\cdot\hat{j}=\hat{k}\cdot\hat{k}=1$$

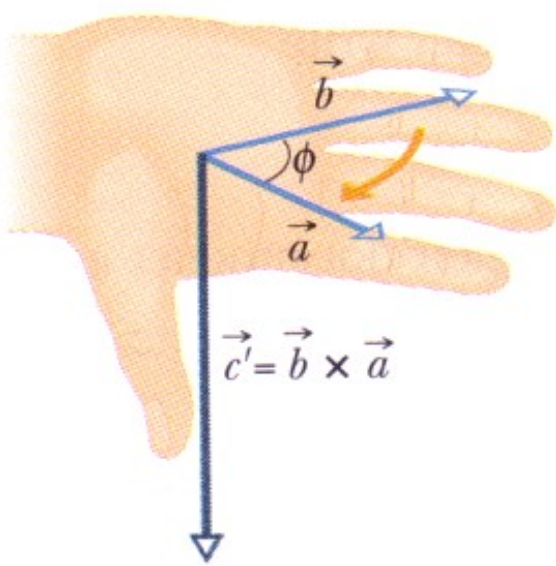
$$\hat{i}\cdot\hat{j}=\hat{j}\cdot\hat{k}=\hat{k}\cdot\hat{i}=0$$

This is the property of orthogonality

The vector product, or cross product



(a)



$$\vec{a} \times \vec{b} = \vec{c}, \text{ where } c = ab\sin\phi$$

$$\vec{a} \times \vec{b} = -\left(\vec{b} \times \vec{a}\right)$$

Direction of $\vec{c} \perp$ to both $\vec{a}$ and $\vec{b}$

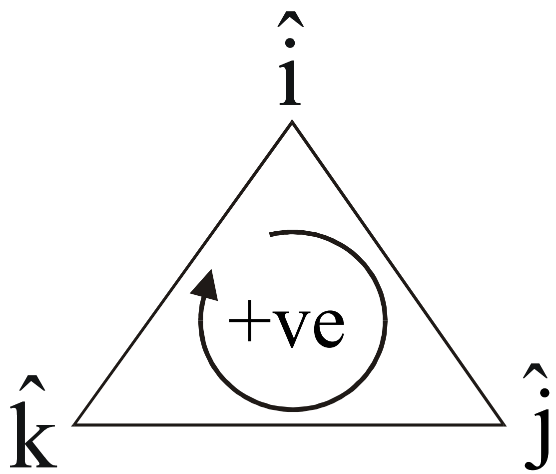
$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

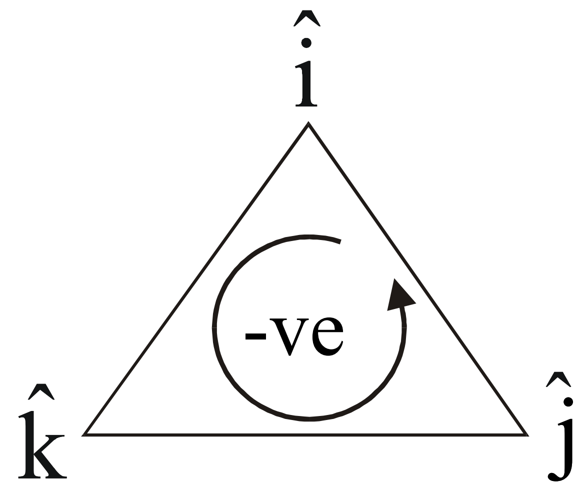
$$\hat{i} \times \hat{j} = \hat{k} \qquad \hat{j} \times \hat{i} = -\hat{k}$$

$$\hat{j} \times \hat{k} = \hat{i} \qquad \hat{k} \times \hat{j} = -\hat{i}$$

$$\hat{k} \times \hat{i} = \hat{j} \qquad \hat{i} \times \hat{k} = -\hat{j}$$

$$\vec{a}\times\vec{b}=\left(a_x\hat{i}+a_y\hat{j}+a_z\hat{k}\right)\times\left(b_x\hat{i}+b_y\hat{j}+b_z\hat{k}\right)$$





$$a_x\hat{i}\times b_y\hat{j}=a_xb_y\left(\hat{i}\times\hat{j}\right)=a_xb_y\hat{k}$$

$$\vec{a}\times\vec{b}=\left(a_yb_z-b_ya_z\right)\hat{i}+\left(a_zb_x-b_za_x\right)\hat{j}+\left(a_xb_y-a_yb_x\right)\hat{k}$$